Math 250 - Notes: Sect. 4.4 - Fundamental Theorem of Calculus

So now you've learned two interpretations of the integral sign (\int). One involves derivatives (or rather, antiderivatives), and the other involves area. Just how these two are related was discovered independently by both Newton and Leibniz, and was so revolutionary that it is called the . . .

FUNDAMENTAL THEOREM OF CALCULUS!

We will develop this important relationship here.

Picture:

Development:

*Fundamental Theorem of Calculus (general form):

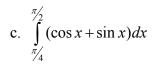
Steps to using the fundamental theorem of calculus to evaluate definite integrals: $\int_{a}^{b} f(x) dx$

- 1. Find the ANTIDERIVATIVE function for *f*.
- 2. Evaluate the antiderivative function first at x = b, then at x = a, and subtract your results.

-examples- Evaluate each definite integral.

a. $\int_{2}^{5} (2x+3)dx$

b.
$$\int_{-1}^{3} (6x^2 - 4x + 1) dx$$



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d. Find the area of the region bounded by the graph of $y = x^2 + 1$, the vertical lines x = -2 and x = 3, and the *x*- axis.

II. Some results from the Fundamental Theorem of Calculus

A. The Mean Value Theorem for Integrals.

If *f* is continuous on the closed interval [*a*, *b*], then there exists a number *c* in the closed interval [*a*, *b*] such that $\int_{a}^{b} f(x)dx = f(c)(b-a)$ This theorem is often written in the form: $f(c) = \frac{1}{b-a} \int_{a}^{b} f(x)dx$, where f(c) is referred to as the "average value of the function."

Picture:

-example- Find the *average value* of $f(x) = 3x^2 - 2x$ on the interval [1, 4].

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-example- Find the value of *c* guaranteed by the mean value theorem for integrals for the function $f(x) = 3x^2 - 2x$ on the interval [1, 4].

B. The Second Fundamental Theorem of Calculus

*Consider the integral $\int_{1}^{x} \cos t \, dt$. Evaluate this.

*Now, take the derivative of your result:

Second Fundamental Theorem of Calculus: If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_{a}^{x}f(t)dt\right] =$$

-example- Evaluate:

a.
$$\frac{d}{dx} \left[\int_{3}^{x} \sin^2 t \, dt \right] =$$

b.
$$\frac{d}{dx} \left[\int_{x}^{2} \tan(4t) dt \right] =$$