Math 250 – Notes: Sect. 4.4 – Fundamental Theorem of Calculus

So now you've learned two interpretations of the integral sign ($\int$). One involves derivatives (or rather, anti-derivatives), and the other involves area. Just how these two are related was discovered independently by both Newton and Leibniz, and was so revolutionary that it is called the . . .

FUNDAMENTAL THEOREM OF CALCULUS!

We will develop this important relationship here.

Picture:

Development:

*Fundamental Theorem of Calculus (general form):

Steps to using the fundamental theorem of calculus to evaluate definite integrals: $\int_a^b f(x)dx$

1. Find the ANTIDERIVATIVE function for *f*.

2. Evaluate the antiderivative function first at $x = b$, then at $x = a$, and subtract your results.

-examples- Evaluate each definite integral.

a. $\int_2^5 (2x+3)dx$

b. $\int_{-1}^3 (6x^2 - 4x + 1)dx$

c. $\int_{\pi/4}^{\pi/2} (\cos x + \sin x)dx$

d. Find the area of the region bounded by the graph of $y = x^2 + 1$, the vertical lines x = -2 and x = 3, and the x- axis.

II. Some results from the Fundamental Theorem of Calculus

A. The Mean Value Theorem for Integrals.

If f is continuous on the closed interval [a, b], then there exists a number c in the closed interval [a, b] such that

$$\int_a^b f(x)dx = f(c)(b-a)$$

This theorem is often written in the form: $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$, where $f(c)$ is referred to as the "average value of the function."

Picture:

-example- Find the *average value* of $f(x) = 3x^2 - 2x$ on the interval [1, 4].

-example- Find the value of c guaranteed by the mean value theorem for integrals for the function $f(x) = 3x^2 - 2x$ on the interval [1, 4].

B. The Second Fundamental Theorem of Calculus

*Consider the integral $\int_1^x \cos t \, dt$. Evaluate this.

*Now, take the derivative of your result:

Second Fundamental Theorem of Calculus: If f is continuous on an open interval I containing a, then, for every x in the interval, $$\frac{d}{dx}\left[\int_a^x f(t)dt\right] =$$

-example- Evaluate:

a. $\frac{d}{dx}\left[\int_3^x \sin^2 t \, dt\right] =$

b. $\frac{d}{dx}\left[\int_x^2 \tan(4t)dt\right] =$